A Gentler Discovery Method
(The Modified Texas Method)

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I believe that the most common mistake made by teachers is that we forget what our principal objective should be. We walk into the classroom saying, "Here is the syllabus that I must cover this semester," forgetting that the need in the classroom is to prepare the student to handle the situations that arise after leaving the classroom. As teachers, we place too much emphasis on the material we "must" cover at the expense of teaching students how to learn on their own. Suppose that a student graduated knowing every fact he was taught. Will these facts be sufficient for success as the student enters the work force or graduate school? Surely not. More likely, the problems the student faces will require learning new material specific to the task at hand and then the application of some of these previously learned facts in conjunction with the new material. The key to whether the student succeeds or fails is his ability to learn on his own.

Mathematics teaching-reform methods fall into one of several categories: introducing technology into the classroom, collaborative learning, and methods such as the Harvard calculus techniques where traditional material is replaced with new material. The method offered here is a gentler type of reform than these controversial methods (Wilson, 1997) being hailed by many reform enthusiasts. It allows the teachers to develop as many or as few new materials as they choose and to involve students interactively in the class as much or as little as is appropriate for the course being taught. These qualities make implementing the method less traumatic for both teacher and student. Specifics are discussed in the section entitled "A Course Description."

The method advocated in this paper is a derivate of the method referred to by the mathematical community as the Texas method or simply Moore's method, after the founder, R. L. Moore, who produced a long list of successful mathematicians (called "Texans") during his tenure at the University of Texas (Moore, 1966). The heart of his technique was to provide students with a very carefully selected sequence of problems and theorems that enabled them to prove and present the material for the class. The modified Moore method, which I use and propose, limits the amount of presentation done by the students to make the method accessible to lower level classes and to facilitate using traditional materials and texts while still emphasizing student presentation. This method fits well with the current reform movements directed toward involving students more actively in the learning process. It prepares students for academia as well as industry by encouraging communication skills, presentation skills, critical thinking (the latest reform buzz-word for "thinking"), and writing skills while alleviating many of the difficulties that we experience as teachers, such as complacent students, students with poor work ethics, (seemingly) low-quality students, and students with low test scores. Further references for the method and the author's exposure to the method are outlined following the body of the paper.

Actively involving the students in the material is fundamental to learning. I defy you to teach a student to ride a bicycle by lecturing at the blackboard.
The student must get on the bicycle and fall. The natural athlete will fall once and then, with a few words from the instructor, successfully ride. The lesser athlete will fall a few times and become frustrated. At this point, the instructor must step in and guide the student, again, through some basic theory of balance, brakes, and pedaling, perhaps demonstrating via example. In a few more attempts, the student will be cycling -- thanks to the teacher and Moore's method.

The method of try, fail, and try again, which is so integral to the Texas method, is illustrated by the famous (among Texans) talk by Dr. M. E. Rudin in which she discussed why she was *not* going to prove a certain theorem. It seems she found a mistake the day before she was to present the paper and proved the converse of what she had stated. Even the best fall off the bicycle occasionally. That is part of learning. We try, we fail, and we try again. It is this fact that makes the Texas method so successful in the classroom. Students work on problems that are difficult, have some measure of success, discuss the problem with the teacher, and attempt the problem again. Does this sound like academic research and research in industry? It should, and it should start in the classroom rather than upon graduation.

Having students present work is one way to cure the problem of emphasizing the syllabus over teaching students to learn. There are a number of advantages to having students put problems on the board. First, the presentation itself is representative of what a student will do in the future. In industry, briefings are standard formats for conveying information, while in academia, presenting papers is an extension of putting problems on the board. Second, the board is a great leveling field; students who may have done quite poorly in other classes (perhaps due to time pressure on exams) are now able to prepare for and present material that they have taken the time to fully understand, thus building some measure of confidence. Third, students learn the necessary mathematical rigor required to convey something that they understand to their classmates. What is clear to the student may not be clear to the audience. In other words, students develop the skill that is the single-most cited attribute that industry seeks when hiring: communication. Finally, the course automatically adjusts to the level of the students. The constant contact with the students alerts me early on to the level of a class. With a weaker class, I will go into less detail in certain areas, while in a stronger class I am able to address additional topics of interest. In many instances, this allows me to alleviate the concern over covering the syllabus that I addressed earlier, since, with a weaker class, I go into less detail and can still complete the syllabus for the course.

Having hailed the benefits of the method, I offer one caveat. I rarely call the method I use the Texas method for two reasons. As a "second generation Texan," I cannot say how closely my methods model those of the late R. L. Moore, and I believe there has been a prejudice against the method in the past. Years ago, I remember a professor saying that he believed there was a move afoot to systematically destroy the method. However, while interviewing for assistant professor positions, I found department heads very open to what I had always considered radical teaching methods. I believe the prejudice stems from the fact that the Texas method is often misunderstood and poorly implemented.

Perhaps I am not qualified to judge the teaching of a man I never met, yet the multitude of classes I took under Texans lead me to certain conclusions. It is my opinion that many of Moore's students believed his method could be described merely by the structure of the course. Problems or theorems are passed out and students present these problems at the board after they solve them. Each class period, volunteers are taken and a pecking order is established. Unless the students are unusually uniform in both mathematical background and talent, this often leads to strong students rising to the top and presenting a majority of the material, while weaker students fall and are subjected to what is essentially a lecture class conducted not by an experienced Ph.D., but rather by other students. If this were Moore's method, then I too am a critic. Yet, I believe this was not his method. I believe Moore to have been as much psychologist as teacher (as most good teachers are), carefully analyzing each student's abilities and providing guidance in such a way that students with a wide range of abilities achieve some measure of success. To clarify further, I describe two Texas classes that I took as a student. I can say without question that these were the two courses in which I learned the least and the most mathematics.

I took the first course as an undergraduate. The class was composed of mathematics education majors, mathematics majors, and a few mathematics graduate students. The class started with about thirty students and by the end of the second quarter (of three) only a few students were presenting material. While the class seemed of little value to a majority of the students, at least one student in that class went on to receive his doctorate quite rapidly. For him, this class had been a stepping stone to higher mathematics, albeit at the expense of the other students. A case could be made that this was a valuable course by that standard. On the other hand, several students from that class, myself included, were steered away from mathematics, at least temporarily, by the experience.

The second course was taken at the graduate level. I can honestly say that through this course, I gained the skills that enabled me to obtain my doctorate. In this class, the range and level of the problems was such that a large percentage of the students put theorems on the board. In fact, out of twenty starting students, at least six went on to receive doctorates. And again the top students in the class excelled and truly used the class as a stepping stone from the mundane regurgitation of mathematics to the sheer beauty of proving theorems and doing mathematics on their own. This time a Texas course had set me on route to my doctorate.

Assuring that a large percentage of the class is presenting on a regular basis may be accomplished by assigning sufficiently many accessible problems for lower-level students, by offering individual help via office hours, and by having students submit problems in written form that may be presented at the board after corrections are made. I have used all of these techniques to level a class of diverse talents. A more controversial technique that I use is to delay the presentation of a problem that a bright student has if no other student has a presentation to offer. This gives the class a day or so to catch up and assures that the course will not become a lecture course "taught" by one or two advanced students. Quite commonly, two students will come by my office on the same day and query me concerning the same problem only to receive very differing amounts of guidance in order to give them a somewhat equal opportunity to present the problem. Using these techniques allows students with a wide range of talents to achieve their personal best in my classes.

The question remains, how does one adapt such methods to classes ranging from freshman college algebra to graduate courses? In (Smith), Michel Smith notes that the method seems considerably more effective at the higher levels. I would argue that this is true of any teaching method. More mature students make for better classes. On the other hand, his calculus course was a true inspiration and starting point for me to develop the modified method that I describe below. Because the traditional Moore methods are quite applicable at higher levels and have been implemented for years with success by teachers too numerous to list here, I offer the description of a lower-level course that I have taught successfully over the past eight years at institutions ranging from Cooke County Community College to Emory University. The method naturally adjusts itself to the level of the students. I would call it a modified Texas method. This strategy has been applied successfully in the following courses: Algebra, Trigonometry, Pre-calculus, Business Calculus, Calculus I, II, Differential Equations, Graduate Ring Theory, and Graduate Numerical Analysis. While I won't say that it went perfectly the first time I tried it, I will say that, from the first attempt, it was clear that it was a method of merit and that the classes were better than in my traditional-style lecture classes.

A COURSE DESCRIPTION:

From freshman to graduate-level courses, I send students to the board. This may take as little as 25% of class time or as much as 95% of class time. Generally, the percentage of time a class spends at the board increases with the level of the class, the exception being when the class has an applied aspect such as numerical analysis. For example, a three-hour-a-week calculus class will spend only one hour a week at the board while a three-hour graduate numerical analysis class will spend about half the class time at the board, and an undergraduate introductory analysis class will spend virtually all of the class time at the board.

Why so varied an amount of time? Because an argument can be made that for certain (prerequisite) courses, having a large body of knowledge on a subject is as important as having a deep understanding of the subject. In my freshman and sophomore level classes, I feel that covering a certain amount of material is a necessity since the students may not get a proponent of the Texas method in the following semester, and lecturing facilitates this goal. In my graduate numerical analysis classes, I feel that I am responsible for covering a wide range of topics so that the students will be well-versed in numerical techniques. Yet in my analysis class, I feel that depth of understanding is crucial at this stage, since this is where students will either learn how to prove theorems or fail to become mathematicians and this class has almost no lecturing.

Grading: The student averages come from three tests (50%), homework presented in class (25%), and a comprehensive final (25%). All problems in my class receive a grade of 0-3 where, 0 = wrong, 1 = mostly wrong, 2 = mostly right, and 3 = right. I like this rather harsh grading system since the student who gets every problem on every test mostly right and receives an "A" on homework has a "C" average. Clearly students strive for better than "mostly right" on their work. Grading flexibility is important, and I admit to students that high homework averages or high final exam grades can raise their final grade above their computed average. At the end of the semester there are inevitably students with "D" averages and "A" on homework. Such a student receives a "C." Also there are students who were bright enough never to come to class for homework days, but have an "A" average on the tests and final. These students receive "A"s, despite an "F" on homework.

Grading flexibility is one of the goals of the method; having interacted with the students all semester, there are inevitably students whose grades do not reflect their mathematical ability. Perhaps they spent too much time exploring alternate ways of solving problems (researching) and not enough time practicing mundane problems. Without the interaction, such students go unnoticed. It has been said that calculus should be a pump, not a filter. This method supports the idea that we should be spending our time catching the good students rather than filtering out the bad.

Structure: I lecture on Monday and Wednesday, assigning problems as I go, and the class presents these problems on Friday. A normal class period has lots of questions because students know that this may be the only source that can help them prepare for the homework I am assigning. There are two definite benefits of assigning problems as I lecture. First, it encourages the students to look over their notes, they must in order to find the homework problems, and second, it assures that I have assigned problems on all the important aspects of a topic. In addition to these problems, I often assign problems from a text. If a student can do every problem I assign at the board, then there is no need to open the text; however, only the brightest of students find this to be the case. The vast majority need practice and they find that practice in the problems I assign from the book. Since I rarely discuss the book, they must decipher other notations. This encourages using the book as a reference rather than a bible. The students need to understand my problems and the book is a possible source of help in doing this. Since I rarely write out my problems beforehand, students must interact with one another to find homework assignments from days they missed. As a consequence, I have good attendance in my classes and good student communication. Cooperative learning would be the reform buzzword.

TGIF! It is Friday and we are ready to go to the board. As I call on students and assign problems for them to present, the first students immediately go the board and put their work up simultaneously. They are allowed (expected) to take their notes and use them. Every problem I assign makes it on the board, so I generally only assign about twenty problems a week and send students to the board five at a time. Each student receives a grade based on the scale previously mentioned.

In truth, I am quite soft on homework grading. For example, if a student does not have the problem they are called on to present, I will let them do another problem of their choice for one point less. Also, I record a grade of
"3" as I call on them and only mark them down for rather serious mistakes, despite what I have told them.

While the students are putting the problems on the board, I answer questions from the students at their seats, but they must vocalize the problem. They may not show me their work. I may also wander the room asking for questions and encourage them to discuss among themselves the mathematics. After the problems are on the board, any students who have questions on the material are encouraged to speak up. I generally ask if they agree with the answers and let students point out mistakes. I don't check every step if the class agrees the problem is right and there are no questions. In fact, I go so far as to tell them that I will not check every step, so they need to in order to verify what is wrong. This way students with incorrect answers will say, "I didn't get that," and we will look more carefully at the problem. Note the self-adjusting nature of the course. If only a few students have the homework, then I am teaching over their heads, while if almost all the students have the work then I can assign harder problems and move faster.

I readily admit that one of my goals in developing this course was my own enjoyment of teaching. I firmly believe that professors who enjoy teaching are good teachers and I look forward to walking into my classroom every day. But I do not enjoy long hours of preparation, grading homework, or grading quizzes. In order to make the class more enjoyable to myself, I want to spend more time interacting with the students and less time on class preparation, grading, and office hours. By lecturing virtually without a book, changes in texts do not add preparation time to the course. Assigning problems as I go requires minimal preparation time as well, although some preparation must be taken to assure they are reasonable problems. Finally, by encouraging questions in class, answering the same question from several students during office hours is minimized. I have found that walking into class without notes or a book catches the students attention. Students are not stupid and they recognize that I am not reading a section and then regurgitating it. Rather, I am guiding and directing the course based on their questions and interests. Because of this they enjoy the class. Because they enjoy the class, they work hard on it. What more can we ask of a class, but that the students enjoy it and learn a lot of mathematics?

I recall the first day I walked into a classroom to teach. After a typical introduction, I started at the board as nervous as I'd been in years. Three-fourths of the blackboard was full and I recall thinking to myself, as I continued to lecture, that I would be teaching at least for the remainder of this semester, probably another, and possibly for the rest of my life. It was time to shed my nerves and relax. I turned to look at my students, only to find them hopelessly nervous and writing frantically. I asked, "How many of you understood what I have said so far?" Two hands went up. I erased the board, told them to tear out the notes they had taken, and started over. I promised I would not erase anything from the board in the future until they understood it -- and do you know what happened? They believed me. My first class was a lot of fun for me, and my willingness to talk with rather than at the students and to alter my course based on their responses has guided my teaching philosophy ever since. The course that I described is a credit to my students' honest responses to the questions I have asked them over the past nine years in person and on evaluations.

Good luck. And remember, interactive teaching is not only good teaching, it is enjoyable teaching.

REFERENCES:

Perhaps the best description of Moore's method is given in his own words (Moore, 1966). Another accurate description of the method is given in (Halmos, 1985). In (Mahavier, 1996), the author describes a graduate applied numerical methods course that utilized a modified Texas method so that the course could include a large quantity of material, including programming, applied mathematics, and pure mathematics. For references concerning upper-level undergraduate courses utilizing the Texas method, consider (Mahavier, in preparation) and (Wilf, 1996). See (Euda, 1996) for descriptions of effective techniques in classes where students learn to prove theorems. (Chalice, 1995) describes a method of addressing the often-cited problem of covering a reasonable amount of material in Texas-method courses by sending many students to the board simultaneously on less difficult problems prior to sending them to the board one at a time for difficult problems.

PERSONAL NOTE and BIOGRAPHY:

My teaching methods are largely a product of the many superb courses I took over the years and my exposure to the Texas method was vast. Michel Smith's calculus class at Auburn University caused my change in major from physics to mathematics. It was my father, W. S. Mahavier, whose examples led me to teach via Moore's method. Paul Lewis taught me more about mathematics and teaching mathematics than anyone, and John W. Neuberger, a man whose infinite optimism is an inspiration to all who know him, directed my doctorate. While I am a firm believer in the method and my education was largely guided by "children" and "grandchildren" of R. L. Moore, H. S. Wall, and H. J. Ettlinger, I remain a strong supporter of other methods, perhaps because a non-Texan, Dean Hoffman, set the stage for my return to graduate school by gently convincing me that I could prove theorems. I feel deeply indebted to my teachers and I repay them in the only way I can, by carrying their examples into my classroom and passing the time they offered me onto my own students. I am currently an Assistant Professor at Nicholls State University, Thibodaux, LA 70310, and may be reached via e-mail at math-wtm@nich-nsunet.nich.edu.

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